

# The Cello as a Model of a Complex Wave System: A Preliminary Research Program

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## Abstract

This note outlines a preliminary research program based on a simple but, I hope, meaningful idea: many physical systems — from a stretched string to a planetary magnetosphere — admit a wave description and therefore call for a common language to analyse their states and regimes. As a first, controllable and physically rich prototype of a complex wave system I propose to use the cello. The aim of the pilot stage is to construct a computational description of the cello's acoustic signal as a “fingerprint” of the underlying wave field of the system, and to formulate a minimal set of quantitative features that can be treated as coarse state variables of this field.

The note does not claim any completed results. Its goal is to formulate the motivation, the research questions, and a realistic plan for a pilot project that can be carried out by an undergraduate student under supervision and later developed into full papers and preprints. On the computational side, the plan assumes a hybrid implementation: low-level signal processing and feature extraction in C++, and statistical analysis, basic machine learning and visualization in Python.

## 1 Introduction and General Motivation

Modern physics provides many examples of systems in which waves and fields play a central role: from acoustics and optics to quantum field theory and astrophysics. Regardless of the nature of the medium (string, plasma, quantum field), the focus is on describing the propagation of perturbations, their spectral content, and possible nonlinear regimes.

At the same time, the development of computational methods and signal analysis makes it increasingly systematic to move from time series  $x(t)$ , measured from real systems, to a set of features and structures that can be interpreted as the *state* and *regime* of a system. For simple linear systems such features are well understood; for more complex, strongly coupled wave systems a common language still exists only in fragments.

The core idea of this project can be stated as follows:

- to treat the **cello** not only as a musical instrument, but as a *controllable complex wave system*, combining vibrations of the string, the body and the air;
- to use this concrete object to develop a set of computational and physical concepts that can later be transferred to other wave systems, up to more astronomical and geophysical problems;

- to start from a careful, limited pilot study in which the main emphasis is on *formulating well-posed questions* and *building a workable analysis*, rather than on maximal model complexity.

In this view, the cello becomes a convenient prototype of a complex wave system: its physics is rich (string–bridge–body–air coupling, nonlinear excitation by the bow), yet it is a system that can be repeatedly and controllably excited and its response measured. This makes it a suitable testbed for developing a language and methods for analysing wave fields.

## 2 Conceptual Framework: Wave Field and Acoustic Signal

### 2.1 The Cello as a Wave System

In the first approximation, a cello string can be described by the one-dimensional wave equation

$$\frac{\partial^2 u(x, t)}{\partial t^2} = c^2 \frac{\partial^2 u(x, t)}{\partial x^2}, \quad c = \sqrt{\frac{T}{\mu}}, \quad (1)$$

where  $u(x, t)$  is the transverse displacement,  $T$  is the string tension,  $\mu$  the linear mass density, and  $c$  the wave speed. The eigenfrequencies of a free string with fixed ends are

$$f_n = \frac{nc}{2L}, \quad n = 1, 2, 3, \dots \quad (2)$$

where  $L$  is the string length. In practice, the situation is much richer: the string is driven by nonlinear bow friction (stick–slip), coupled via the bridge to the body, and further to the acoustic field in the air.

Schematically, the physical system can be represented as a chain of coupled subsystems:

bow  $\longrightarrow$  string  $\longrightarrow$  bridge  $\longrightarrow$  body + air  $\longrightarrow$  microphone.

At each stage the system has its own pattern of oscillations and its own effective impedance for energy transfer. Ultimately, the microphone records an acoustic signal  $x(t)$  which is a complicated superposition of the body modes of the instrument and the acoustic field in the surrounding air.

### 2.2 The Signal as a Projection of the Wave Field

Within the proposed approach, the acoustic signal  $x(t)$  is viewed as a *projection* of a high-dimensional state of the wave field onto a one-dimensional measurement (pressure or particle velocity near the microphone). The task of analysis is to associate this signal with a compact feature vector

$$\Phi[x] = (F_1[x], F_2[x], \dots, F_m[x]), \quad (3)$$

which can be interpreted as coarse, effective state variables of the system:

- spectral energy in given frequency bands;
- harmonic structure (frequencies  $f_n$  and amplitudes  $A_n$ );

- integral spectral descriptors (spectral centroid, bandwidth, spectral slope);
- if necessary, more complex time–frequency features (spectral flux, instability measures, etc.).

Throughout this note,  $\Phi[x]$  is intended as a coarse, robust description of the state of the system, not as a full microscopic model of the underlying wave field. Intuitively, the goal is to reach a situation in which different regimes of the wave field (roughly: “clean sustained tone”, “noisy, breaking regime”, “anomalous oscillations”) occupy well-separated regions in the feature space  $\mathbb{R}^m$ . In that case,  $\Phi[x]$  can be treated as a set of *state coordinates* in a simplified phase space, albeit a very coarse one.

### 3 Research Questions for the Pilot Stage

To connect the general motivation with concrete, feasible work, it is helpful to formulate several simple but sharp questions that one can attempt to answer using real measurements and elementary physics. For the pilot stage I propose the following three questions.

#### Question 1. Why does the cello sound different from the violin and the double bass?

Even a non-musician can hear that the cello sounds fundamentally different from the violin or the double bass, despite all three being bowed string instruments with four strings and a wooden body. The intuitive answer “because of size and construction” is too vague and gives little in terms of quantitative description.

**Question 1.** *Can a small set of physically meaningful features extracted from the acoustic signal (distribution of energy across frequencies, structure of the first harmonics, spectral slope, etc.) clearly demonstrate how the sound of the cello is “organized” compared to the violin and the double bass?*

Practically, this means:

- choosing 5–10 simple spectral and energy descriptors;
- recording isolated tones (e.g. open strings) on all three instruments under as similar conditions as possible (same microphone, fixed distance and placement, comparable dynamics and room acoustics);
- checking whether these features form stable, distinguishable “fingerprints” for each instrument.

The aim is not a full acoustic model, but an *elementary, yet honest* explanation of how the cello sound is quantitatively different from its bowed relatives.

#### Question 2. A Minimal Set of State Coordinates for a Single Cello

If we focus on a single cello, the natural next step is to look for a minimal set of numbers that can be interpreted as coarse “state coordinates” of its wave field.

**Question 2.** *Is there a small, noise-robust set of quantitative features  $\Phi[x]$  extracted from the signal of one cello, which (a) distinguishes qualitatively different playing regimes*

(stable sustained tone, intentionally “harsh” sound, nearly breaking regime), and (b) is relatively insensitive to secondary variations of performance?

Here compactness is important: the goal is that a few parameters (for example, relative amplitudes of the first harmonics, spectral centroid, energy in a high-frequency band) already allow one to speak about the state of the system in physical terms, without turning the analysis into a collection of hundreds of loosely interpretable indices.

### Question 3. The Structure of the Regime Space and Transitions Between Regimes

Once each recorded sound is associated with a feature vector  $\Phi[x]$ , the whole dataset can be viewed as a cloud of points in a multidimensional space. The question is whether this space has a meaningful structure.

**Question 3.** *Do different ways of playing the cello (stable tone, “surface” noisy regime, deliberately overloaded sound, etc.) form distinguishable clusters in the feature space  $\mathbb{R}^m$ , and can these clusters be interpreted as different regimes of the underlying wave system?*

In practice, this entails:

- recording a limited but controlled set of sounds of the *same* note in different regimes;
- computing the chosen set of features for each sound;
- visualizing and analysing the resulting point cloud (for example, via projections onto the first 2–3 principal components).

Of additional interest is the behaviour of the system under smooth changes of playing parameters (e.g. gradual increase of bow force or gradual shift of the contact point). Trajectories in feature space may show how the system moves from one regime to another and where the boundaries of stability lie.

## 4 Pilot Project Plan

What follows is not a detailed week-by-week schedule, but a logical sequence of steps, each of which, in my view, can be realistically completed within one short-semester (6 weeks) under reasonable workload.

### 4.1 Step 1. Refining the Physical and Computational Quantities

At this stage the tasks are:

- to refine the list of features  $\Phi[x]$  to be used in the first stage (e.g. 5–10 quantities);
- to tie each feature to physical intuition: how it relates to the distribution of energy across modes, to perceived “brightness”, to the presence or absence of a stable fundamental, etc.;
- to ensure that all selected quantities can be computed correctly and reliably from a finite, limited-length discrete signal.

## 4.2 Step 2. Collecting a Limited but Controlled Dataset

The plan is to record a set of cello sounds in relatively simple but controlled conditions:

- one cello, one player (the author of the project);
- several fixed pitches (e.g. open strings and a few stable stopped notes);
- several qualitatively different playing regimes (stable tone, intentionally “harsh” sound, deliberately unstable bowing, etc.);
- repetitions: several realizations of each regime;
- controlled recording setup: same microphone and preamp, fixed distance and angle, and as stable as possible dynamics and room conditions across all sessions.

The goal is not an exhaustive mapping of all possibilities of the instrument, but to obtain sufficiently clean material on which features can be selected and tested.

## 4.3 Step 3. Computational Analysis Pipeline (C++ + Python)

I propose to build a hybrid pipeline:

- in C++: basic processing — reading WAV files, selecting quasi-stationary segments, computing spectra (FFT), extracting the chosen features;
- in Python: statistical analysis and visualization — comparing feature distributions across regimes, searching for clusters, building simple models (e.g. linear discriminants between regimes).

On the data analysis side, the plan is to begin with standard tools from basic machine learning. In particular:

- Principal Component Analysis (PCA) to obtain low-dimensional projections of the feature cloud and to identify dominant directions of variation;
- simple clustering algorithms such as  $k$ -means, and, if feasible, Gaussian mixture models trained via the EM algorithm, to test whether different playing regimes naturally form separate clusters in feature space.

These methods are not the focus of the project by themselves, but provide a convenient way to summarize and visualize the structure of the data generated by the physical system.

This step serves both as training in computational methods and as a test of the overall feasibility of the approach. The separation of tasks reflects a natural division of labour: efficient low-level signal processing in C++, and rapid exploratory data analysis using the Python ecosystem (NumPy, pandas, matplotlib, scikit-learn, etc.).

## 4.4 Step 4. Interpretation and Formulation of Higher-Level Programs

At the end of the pilot the goals are:

- to identify which features are truly informative and which are redundant or too noisy;

- to understand how naturally the differences between regimes can be interpreted in feature space;
- to formulate the next level of questions: inclusion of transient processes, explicit nonlinear effects, extension to other classes of wave systems.

The outcome of this step is intended to be a first self-contained research article or preprint (for example, suitable for submission to arXiv), presenting the experimental setup, the feature-extraction pipeline, and quantitative comparisons between different playing regimes. In parallel, I hope that some of the intuition and analysis methods developed on the cello prototype will later carry over, at least at the level of concepts and tools, to more general wave fields (plasma, seismology, magnetospheric processes, etc.).

## 5 Expected Contribution and Future Development

Even in its most modest variant, the pilot project described above should provide:

- a working example of how a real complex wave system (the cello) can be mapped into a space of quantitative features that admit physical interpretation;
- an understanding of the limitations of such a description: in which situations simple spectral features are sufficient, and where more sophisticated models are required;
- a possible foundation for a broader program in which a musical instrument becomes a stepping stone toward the analysis of other wave systems, from laboratory settings to astronomical and geophysical environments, at least on the level of shared intuition and analysis methods.

The proposed program is intended as a starting point: at the first stage, basic concepts and computational tools are tested on an accessible object; then this language can be extended to more complex wave systems and more physics-rich problems.