About this document

This document is an extended analytical companion to the main research paper *The Cello as a Model of a Complex Wave System: Feature Extraction, Phase Space, and Stability Transitions*. The main manuscript presents the central model, feature-extraction pipeline, and quantitative results. This expanded text develops the deeper theoretical background of the project: the physics of bowed-string dynamics, Helmholtz stability, transient response, friction and playability models, geometric transition detection in reduced feature space, and the broader transferability of the framework to other nonlinear wave systems.

It is provided for readers who would like to go beyond the compact research article and explore the wider mathematical, physical, and conceptual structure of the project.

Analytical Foundations of Coupled Wave Systems: The Cello as a Controllable Prototype for Multi-Regime Stability and Cross-Disciplinary Transition Detection

The study of complex wave systems has long been hindered by the inherent difficulty of identifying precise state variables and control parameters in nonlinear, coupled environments. Traditional approaches often isolate individual components—such as a single vibrating string or a localized plasma perturbation—thereby neglecting the essential interactions that define the system's global behavior. However, the cello has recently emerged as a singular, controllable prototype for the study of these coupled systems, offering a rich environment where string vibration, body resonance, bridge coupling, and nonlinear acoustic radiation intersect in a measurable and reproducible manner.[1] By abstracting the cello into a mathematical model of a coupled wave system, it becomes possible to develop a computational pipeline for state extraction that is not only robust within the domain of musical acoustics but also transferable to high-stakes systems in geophysics, magnetospheric physics, and helioseismology.[1]

At the heart of this multidisciplinary inquiry is the transition from coherent, periodic motion to chaotic or raucous regimes. In the context of a bowed string, this transition is governed by the nonlinear interaction between the bow and the string, a process that mirrors the stick-slip dynamics observed in seismic fault lines and the field line resonances of magnetospheric plasma.[1] Recent research has demonstrated that a 12-dimensional feature vector, derived from the physical equations of bow-string interaction, can form stable "fingerprints" for both instrument identity and playing regimes, allowing for the detection of stability transitions through the geometric analysis of trajectories in a reduced principal component space.[1]

The Physics of Bowed-String Dynamics and Helmholtz Stability

The fundamental oscillator of the cello system is a string of length L, tension T, and linear density μ. The transverse displacement $u(x,t)$ is governed by the classical one-dimensional wave equation, where the wave speed c is defined as $\sqrt{T/\mu}$.[1] The introduction of the bow, however, transforms this linear system into a strongly nonlinear one. The bow applies a force $F_b(t)$ at a contact point x_b, which is modeled using a velocity-weakening friction law where the effective friction coefficient μ_{eff} is a function of the relative velocity v_{rel} between the bow and the string.[1]

Idealized Helmholtz Motion and Harmonic Scaling

Under optimal stick-slip conditions, the string adopts a regime known as Helmholtz motion. This motion is characterized by a "corner" that travels in a V-shape along the string, triggering the transition between sticking and slipping as it passes the bow.[7] The resulting velocity waveform at the bridge follows a sawtooth pattern, which can be represented by a Fourier series where the amplitudes A_n of the n-th harmonics are inversely proportional to n.[1]

In practical applications, the body impedance of the instrument modifies this idealized relationship. Analysis of different members of the violin family shows that the spectral roll-off follows a power law $A_n \propto n^{-\gamma}$, where γ serves as a structural indicator of the instrument's size and acoustic damping characteristics.[1]

Instrument Type	**Fundamental Frequency (f0)**	**Roll-off Exponent (γ)**	**Noise Parameter (σ)**
Violin	196.0 Hz	0.70	0.04
Cello	65.4 Hz	1.10	0.06
Double Bass	41.2 Hz	2.00	0.05

The distinct γ values—0.70 for the violin, 1.10 for the cello, and 2.00 for the double bass—represent the physical manifestation of body impedance filtering, providing a primary axis for instrument classification in feature space.[1]

Schelleng Bounds and the Window of Stability

The sustainability of Helmholtz motion is constrained by specific bounds on the normal bow force F_N, first derived by Schelleng. These bounds define the range within which the bow can maintain the stick-slip cycle without either slipping prematurely or failing to release.[1] The minimum and maximum forces are given by:

$$F_N^{min} = \frac{2\pi Z_0 v_b}{\mu_s - \mu_k}\beta_b$$

$$F_N^{max} = \frac{Z_0 v_b}{2(\mu_s - \mu_k)\beta_b}$$

where Z_0 is the characteristic impedance of the string, v_b is the bow velocity, and $\beta_b = x_b/L$ is the relative bowing position.[1] The ratio between the maximum and minimum force, $\pi/(4\beta_b^2)$, illustrates that the stability window narrows as the bow moves closer to the bridge, making the system increasingly sensitive to small perturbations in playing parameters.[1] This narrowing of the stability window is a critical feature of all coupled wave systems where the control parameter is applied near a boundary.[1]

Feature Engineering and the 12-Dimensional State Vector

To extract the state of the system from its acoustic observable $x(t)$, a pipeline has been developed that maps the signal to a 12-dimensional feature vector $\Phi[x]$. Each component of this vector is grounded in the underlying physics of the string-body interaction.[1]

Spectral and Harmonic Feature Construction

The first seven components of the feature vector are the normalized harmonic amplitudes H_2 through H_8, calculated as $|X(nf_0)|/|X(f_0)|$. These capture the energy distribution within the harmonic series, which is directly influenced by the stick-slip regime and the body's spectral response.[1] Beyond simple harmonicity, the vector incorporates features that characterize the "shape" of the spectrum:

- **Spectral Centroid (ν_c):** This value represents the "center of mass" of the spectrum and is highly sensitive to the high-frequency excitation that occurs as the system approaches a raucous or chaotic state.[1]
- **Bandwidth ($\Delta\nu$):** This measures the spectral spread around the centroid, providing an indicator of the signal's coherence.[1]
- **Spectral Slope (α):** Characterized in dB/kHz, the slope describes the rate of spectral decay, which is a function of both the string's internal damping and the body's radiation efficiency.[1]

- **High-Frequency Energy Ratio (ρ_{HF}):** This captures the proportion of total energy present above 2 kHz, a region that becomes noise-dominated in the near-breaking regime.[1]
- **Inharmonicity (η):** Derived from the modified eigenfrequency equation $f_n = nf_1\sqrt{1+\beta_{inh}n^2}$, where β_{inh} depends on string stiffness and tension. This feature provides a pitch-independent fingerprint for the specific string being analyzed.[1]

The inclusion of inharmonicity is particularly vital for distinguishing between different string types on the same instrument, as variations in bending stiffness can significantly alter the "playability" and the resulting transient response.[11]

Manifold Analysis and Dimensionality Reduction

The 12-dimensional feature space provides a comprehensive description of the system's state, but its high dimensionality complicates the identification of regime transitions. Principal Component Analysis (PCA) serves as a necessary step for unravelling the hidden structures within the data.[1]

Inter-Instrument Variance and PCA Loadings

When comparing different instruments (violin, cello, and double bass), PCA reveals that the effective dimensionality of the system is extremely low. Two principal components explain 98.8% of the total variance.[1]

Component	**Explained Variance (%)**	**Primary Physical Drivers**
PC 1	78.5%	Spectral Centroid, HF Energy Ratio (Brightness Axis)
PC 2	20.3%	Harmonic Amplitudes H_2, H_3, H_4 (Richness Axis)
Cumulative	**98.8%**	**Total System Variance**

The dominance of PC1 suggests that the primary discriminator between bowed instruments is their high-frequency response, which is a direct consequence of body size and its impact on the roll-off exponent γ. PC2, meanwhile, captures the nuances of harmonic richness dictated by body damping and coupling efficiency.[1] In this reduced 2D space, k-means classification achieves a perfect Adjusted Rand Index (ARI) of 1.000, confirming that the structural parameters of these instruments form non-overlapping clusters.[1]

Intra-Instrument Regime Variance

In contrast to the clear separation between instruments, identifying qualitative playing regimes within a single cello is a more complex task. Stick-slip dynamics are inherently nonlinear and non-stationary, meaning the correlation structure of the features evolves as the control parameter changes.[1] Within a single instrument, PC1 and PC2 account for only 48.1% of the variance, and three components are required to capture 68.2%.[1]

This higher intrinsic dimensionality reflects the fact that a change in bow force simultaneously modifies multiple physical variables, including the noise floor σ, the spectral roll-off γ, and the intensity of high-mode excitation.[1] The boundary between "harsh" sounds and "near-breaking" regimes is not a discrete threshold but a continuous gradient, leading to a lower k-means ARI of 0.540.[1] This result is a direct computational confirmation of the Schelleng physics, which predicts that the transition from Helmholtz motion to chaotic vibration occurs over a finite window of force rather than at a sharp wall.[1]

Analytical Trajectory Curvature and Transition Detection

The central breakthrough in this research is the development of a model-free method for locating stability transitions using the geometric properties of feature space trajectories.[1] By sweeping the bow force parameter β continuously from 0 to 1, the system's state traces a smooth path through the principal component space.

The Curvature Maximum Signature

The trajectory curvature $\kappa(\beta)$ is calculated as the rate of change of the tangent vector along the PC-reduced manifold.[19] The analysis shows that κ reaches its maximum at $\beta \approx 0.77$, a point that identifies the physical stability transition without any prior labeling of the training data.[1]

This curvature maximum indicates a reorganisation of the system's internal dynamics. At low values of β, the trajectory is dominated by changes in the harmonic richness axis. However, as β approaches the critical threshold, the noise-sensitive features—specifically the spectral centroid and bandwidth—begin to vary more rapidly than the harmonic ratios.[1] The "kink" in the trajectory thus serves as a mathematical signature of the transition from coherent, stick-slip Helmholtz motion to noise-dominated, raucous dynamics.[1] This geometric approach avoids the subjectivity of perceptual labels like "harsh" or "breaking" and provides an objective metric for stability analysis in any system admitting a scalar observable and a control parameter.[1]

Advanced Nonlinear Dynamics of the Transient Response

While steady-state motion provides a stable point for analysis, the transient response—the behavior of the string at the beginning of a note—holds the key to understanding the system's sensitivity to initial conditions.[6]

Multiple Helmholtz Waves and Stick-Slip Evolution

Recent numerical studies focusing on the transient response have moved beyond the assumption of a single corner motion. Simulations utilizing discretized masses connected by springs have confirmed that multiple Helmholtz waves can be observed during the initial attack phase.[6] These waves can be approximately separated into individual components, allowing researchers to track the evolution from a chaotic onset to a unified steady state.[6]

The Rise to Steady State is characterized by:

1. **Early Regular Triggering:** The process of establishing a periodic slip-stick cycle within the first few periods of vibration.[6]
2. **Wave Amalgamation:** The merging of multiple initial disturbances into a single, dominant Helmholtz corner.[6]
3. **Hysteresis in Force Reconstruction:** The observation that friction force is highest at the start of slipping and lower on the return to the sticking phase, which is a consequence of the thermal inertia of the rosin coating.[21]

Understanding these transient dynamics is essential for explaining the "speckled" nature of playability maps, where successful attacks are interspersed with failed ones even when control parameters are nominally within the Schelleng bounds.[11]

Elasto-Plastic Friction and Rosin Tribology

The most advanced models of the bow-string interaction have shifted away from simple static friction curves toward complex elasto-plastic formulations.[21] These models provide a more accurate representation of the rosin-mediated contact between the bow hair and the string.

The Bristle Model and Breakaway Thresholds

In these formulations, the contact is modeled as an ensemble of elastic bristles. Each bristle contributes to the total friction load until it reaches a specific breakaway displacement.[23]

Sliding Regime	Displacement Condition	Phenomenological Behavior
Pre-sliding (Sticking)	$	z
Mixed Elasto-Plastic	$z_{ba} <	z
Purely Plastic (Slipping)	$	z

A critical refinement in the 2025 research is the derivation of a stable numerical scheme that guarantees **passivity**.[21] This ensures that the simulation inherits the energy balance of the underlying continuous model, preventing the non-physical growth of energy that plagued earlier discrete models.[25]

Thermal Effects and Viscosity Fluctuations

A major breakthrough occurred with the discovery that contact temperature is the primary state variable for rosin friction.[21] Rosin is a glassy material that transitions to a liquid state as the sliding velocity—and thus the heat generated by friction—increases.[23] The resulting melt lubrication explains the falling friction-velocity characteristic: as the temperature of the rosin layer rises, its viscosity drops, reducing the resistance to sliding.[22]

Hysteresis arises naturally in this thermal model because the rosin's temperature lags behind changes in sliding speed due to thermal inertia.[21] This refined "thermal elasto-plastic" model bridges the gap between steady-state predictions and transient reconstructions, providing a physical explanation for why the friction force is lower on the way back toward the sticking phase than it was on the way out.[21]

Experimental Validation and the Guettler Diagram

The theoretical limits of stability are traditionally visualized using the Guettler diagram, which plots transient time within a two-dimensional space defined by **bow force** and **bow acceleration**.[11]

Robotic Bowing and Playable Wedges

Experimental investigations using robotic arms (such as the UR5e) have provided a rigorous test of Guettler's conditions.[8] These experiments collect data at high sampling rates (50 kHz) to extract "pre-Helmholtz" transient times—the duration between the first slip and the first instance of sustained Helmholtz motion.[12]

Four primary conditions (A-D) govern the "perfect attack" within this diagram:

- **Condition A (Upper Limit):** Ensures the bow force is sufficient to prevent premature slipping before the reflected wave returns from the nut.[10]
- **Condition B (Lower Limit):** Ensures the force is small enough to allow the wave to pass the bowing point during the slip phase.[10]
- **Condition C & D:** Address the sustaining of the cycle during the time-evolution of the friction force.[12]

Experimental results confirm the existence of a triangular "playable wedge" that rotates clockwise as the bow-bridge distance β increases.[12] However, the diagrams also reveal "speckled" distributions of failed transients within the wedge, suggesting that factors like string inharmonicity, damping at the supports, and instrument response play a more significant role than predicted by idealized friction models.[11]

Cross-Disciplinary Transferability: Structural Analogies in Wave Physics

The computational pipeline developed for the cello is founded on structural conditions that are ubiquitous across physical scales: a scalar observable, a spectral structure that evolves with the system's state, and a control parameter.[1]

Magnetospheric Plasma and Alfvén Waves

The cello wave equation is mathematically equivalent to the Alfvén wave equation, which describes the propagation of magnetic field disturbances in an electrically conducting fluid.[1] In this analogy, magnetic field lines act as the "strings," and their tension is provided by the magnetic pressure $B_0^2/(2\mu_0)$.[3]

The "bowing" mechanism in plasma is represented by shear flows or the solar wind, which can "pluck" or "bow" the field lines through a relative motion that mirrors stick-slip dynamics.[28] Just as a cello string transitions from coherent Helmholtz motion to raucous noise, field line resonances (FLRs) can transition to broadband hiss.[1] Ground magnetometer data provides the scalar observable, while solar wind pressure serves as the control parameter.[1] The transition detection pipeline could thus be used to identify the onset of plasma instabilities in the Earth's magnetosphere before they manifest as global state shifts.[1]

Seismic Tremors and Rate-and-State Friction

The Woodhouse friction model used in bowing is structurally identical to the rate-and-state friction laws developed by Dieterich and Ruina for geological faulting.[1] Both systems exhibit stick-slip instabilities where the "stick" phase corresponds to the accumulation of elastic strain energy and the "slip" phase corresponds to a sudden release of that energy—an earthquake or a string release.[2]

The "slowness law" in seismology, which describes the logarithmic growth of static friction over time, is the geological analogue to the cooling and restrengthening of rosin between bow slips.[2] The transition from seismic tremor (analogous to Helmholtz motion) to a full earthquake nucleation event (analogous to the raucous regime) involves a reorganisation of features that can be detected via trajectory curvature in seismic data.[1]

Helioseismology and Stochastic Excitation

Solar p-modes, the acoustic normal modes of the Sun, are driven by turbulent convection—a stochastic excitation mechanism.[35] This is analogous to the "noisy bow" of the near-breaking cello regime, where the bow's stochastic slips drive the resonant modes of the instrument body.[1]

By analyzing the Doppler velocity of the photosphere (the scalar observable) as a function of solar wind pressure (the control parameter), the curvature-based transition detection method can be applied to helioseismic data to identify precursors to changes in the Sun's convective regime.[1]

Manifold Learning and Nonlinear Dimensionality Reduction

The success of PCA in these analyses relies on the Manifold Hypothesis, which suggests that high-dimensional physical signals often lie on a lower-dimensional latent manifold.[13] However, the linearity of PCA means that it may struggle to capture the complex, intertwined patterns found in highly non-stationary systems.[13]

Sammon Mapping, Isomap, and Geodesic Distances

When the feature space is strongly curved—as it is during a stability transition—nonlinear manifold learning algorithms may offer superior insight.[13]

- **Sammon Mapping:** This technique minimizes an error function (Sammon's stress) to preserve the overall topology of the data, ensuring that points distant in the original 12D space remain distant in the reduced 2D projection.[13]
- **Isomap:** Unlike PCA, which uses Euclidean distance, Isomap calculates **geodesic distances** along the manifold by constructing a neighborhood graph.[13] This is particularly useful for identifying the "true" distance between different playing regimes that might appear close in a linear projection but are separated by a curved path of physical

transitions.[13]

- **t-SNE and UMAP:** These techniques are optimized for visualizing distinct clusters, such as different instrument types or stable vs. unstable regimes. UMAP, in particular, is noted for its ability to balance global and local structure, making it faster and more consistent than t-SNE for tracking long-term state trajectories.[13]

In the case of the cello, the trajectory in feature space is effectively a rank-1 manifold (a one-dimensional curve) that "bends" during the stability transition.[15] PCA snapshots are sufficient to capture the curvature of this rank-1 manifold, but UMAP or Neurospectrum frameworks—which incorporate spatial and temporal structure—could provide even more nuanced warnings of impending state reorganisation.[1]

Potential Formulation for Rate-and-State Friction

A final piece of the analytical puzzle is the development of a potential formulation for friction laws. Rate-and-state friction, by its nature, lacks a traditional potential or variational formulation, making implicit time discretization computationally expensive and unstable.[31]

Deep Learning and Variational Approaches

Recent work (2025) has proposed formulating these potentials as neural networks trained to emulate empirical rate-and-state behavior.[31] By representing history-dependent phenomena through recurrent neural operators (RNOs), researchers can learn the evolution of state variables directly from the data.[31]

This approach allows for:

1. **Convex Variational Problems:** Enabling stable implicit time discretization and more efficient numerical simulations of stick-slip events.[31]
2. **History-Dependent Surrogates:** Replacing expensive small-scale physical models with neural surrogates that can be used in large-scale seismic or acoustic simulations.[44]
3. **Accuracy Reconciliation:** These formulations have been shown to reproduce the behavior of empirical laws with errors smaller than those found in experimental data matching.[31]

The application of these variational friction potentials to the bowed string could finally allow for real-time, high-fidelity simulations that accurately co-predict steady-state and transient behavior across all musical ranges.[21]

Synthesis and Strategic Outlook

The analysis of the cello as a controllable coupled wave system has yielded a computational framework that transcends the boundaries of musical acoustics. The demonstration that a high-dimensional feature vector can be condensed into a 2-dimensional space—preserving 98.8% of structural variance—marks a significant milestone in the objective classification of physical systems.[1] Furthermore, the discovery that the geometric curvature of the state trajectory provides a model-free warning of stability transitions offers a powerful new tool for predictive maintenance and hazard forecasting.[1]

The future of this research lies in the validation of the PCA-curvature pipeline against real-world cello recordings and its subsequent deployment across target systems:

1. **Magnetospheric Monitoring:** Utilizing magnetometer arrays to detect FLR reorganisation as a precursor to plasma storms.[1]
2. **Seismic Hazard Assessment:** Applying high-frequency spectral analysis to fault-line tremors to identify the shift from stable creep to earthquake nucleation.[1]
3. **Engineering Stability:** Implementing stick-slip detection in mechanical joints and turbine blades to prevent catastrophic failure through raucous-regime identification.[32]

By reading the "curvature of the state," we gain the ability to intervene in complex systems before they reach the point of failure. The cello, once viewed merely as a tool for artistic expression, has now become a cornerstone for the unified study of nonlinear waves, providing the conceptual and computational foundation for a new era of proactive physical analysis.[1]

Works cited

1. chello_research.pdf
2. Earthquakes and friction laws - Active Tectonics, Quantitative Structural Geology and Geomorphology, accessed May 12, 2026, http://activetectonics.asu.edu/ActiveFaultingSeminar/Papers/Scholz_1998.pdf
3. Alfvén wave - Wikipedia, accessed May 12, 2026, https://en.wikipedia.org/wiki/Alfv%C3%A9n_wave
4. Wave equation - Wikipedia, accessed May 12, 2026, https://en.wikipedia.org/wiki/Wave_equation
5. On the fundamentals of bowed string dynamics - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/247040060_On_the_fundamentals_of_bowed_string_dynamics
6. (PDF) Numerical study on transient response of the bowed string, accessed May 12, 2026, https://www.researchgate.net/publication/390210145_Numerical_study_on_transient_response_of_the_bowed_string
7. The Bowed String As We Know It Today - Euphonics, accessed May 12, 2026, https://euphonics.org/wp-content/uploads/2022/03/BowedStringReview.pdf
8. ON THE PLAYABILITY OF STRINGED INSTRUMENTS, accessed May 12, 2026, https://www.repository.cam.ac.uk/bitstreams/68830e1d-c0d7-4fb1-b88b-7cbe754d653e/download
9. Simulation of the effects of tilting the bow; bow flat on string... | Download Scientific Diagram - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/figure/Simulation-of-the-effects-of-tilting-the-bo

w-bow-flat-on-string-middle-bow-tilted-in_fig1_253603633
10. On the Creation of the Helmholtz Motion in Bowed Strings - Knut's Acoustics, accessed May 12, 2026, https://knutsacoustics.com/files/actaacustica-on-the-creation.pdf
11. An experimental approach for comparing the influence of cello string type on bowed attack response - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/385645515_An_experimental_approach_for_comparing_the_influence_of_cello_string_type_on_bowed_attack_response
12. Assessing playability limits of bowed-string transients using ..., accessed May 12, 2026, https://acta-acustica.edpsciences.org/articles/aacus/full_html/2024/01/aacus240063/aacus240063.html
13. No Straight Lines Here: The Wacky World of Non-Linear Manifold ..., accessed May 12, 2026, https://sites.gatech.edu/omscs7641/2024/03/10/no-straight-lines-here-the-wacky-world-of-non-linear-manifold-learning/
14. Principal component analysis - Wikipedia, accessed May 12, 2026, https://en.wikipedia.org/wiki/Principal_component_analysis
15. Low-Dimensional and Transversely Curved Optimization Dynamics in Grokking - arXiv, accessed May 12, 2026, https://arxiv.org/html/2602.16746v3
16. Nonlinear dynamics of the wolf tone production | Request PDF - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/354742221_Nonlinear_dynamics_of_the_wolf_tone_production
17. Sensitivity of principal components to changes in the presence of non-stationarity - arXiv, accessed May 12, 2026, https://arxiv.org/pdf/2208.04651
18. A Grain Growth Study: the Five Parameter Grain Boundary Curvature Distribution and the Evolution of Grain Face Area - Carnegie Mellon University, accessed May 12, 2026, http://mimp.materials.cmu.edu/rohrer/theses/190927_Thesis_xiaoting_zhong.pdf
19. Local Invariant Feature Detectors: A Survey | Foundations and Trends in Computer Graphics and Vision | Emerald Publishing, accessed May 12, 2026, https://www.emerald.com/ftcgv/article/3/3/177/1319342/Local-Invariant-Feature-Detectors-A-Survey
20. Identifying short-term variation of dynamic friction by means of its frequency response function - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/338893974_Identifying_short-term_variation_of_dynamic_friction_by_means_of_its_frequency_response_function
21. Ewa Matusiak's research works | University of Music and Performing Arts Vienna and other places - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/scientific-contributions/Ewa-Matusiak-2262035280
22. Tribology of rosin | Request PDF - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/239354022_Tribology_of_rosin
23. A refined bow-string interaction model considering hysteresis - Queen's

University Belfast Research Portal, accessed May 12, 2026, https://pure.qub.ac.uk/files/664948977/poma.pdf
24. Elasto-plastic friction modeling of bow–string interaction | Request PDF - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/394661845_Elasto-plastic_friction_modeling_of_bow-string_interaction
25. Numerical modelling of elasto-plastic friction in bow-string interaction with guaranteed passivity - Queen's University Belfast, accessed May 12, 2026, https://pureadmin.qub.ac.uk/ws/portalfiles/portal/640362595/alkji.pdf
26. Numerical modelling of elasto-plastic friction in bow–string interaction with guaranteed passivity - mdw, accessed May 12, 2026, https://pub.mdw.ac.at/media/content_files/frsip-2-1525044.pdf
27. Numerical modelling of elasto-plastic friction in bow-string interaction with guaranteed passivity - Frontiers, accessed May 12, 2026, https://www.frontiersin.org/journals/signal-processing/articles/10.3389/frsip.2025.1525044/abstract
28. US7777120B2 - Electromagnetic musical instrument frequency conversion systems and related methods - Google Patents, accessed May 12, 2026, https://patents.google.com/patent/US7777120B2/en
29. arXiv:1603.08205v1 [math.AP] 27 Mar 2016, accessed May 12, 2026, https://arxiv.org/pdf/1603.08205
30. Perspective Chapter: Numerical Simulation to Study Alfvén Waves - IntechOpen, accessed May 12, 2026, https://www.intechopen.com/chapters/1141263
31. Learning a potential formulation for rate-and-state friction - arXiv, accessed May 12, 2026, https://arxiv.org/html/2507.09796v1
32. Are there reliable constitutive laws for dynamic friction? - Royal Society Publishing, accessed May 12, 2026, https://royalsocietypublishing.org/rsta/article/373/2051/20140401/114947/Are-there-reliable-constitutive-laws-for-dynamic
33. Dynamic friction in sheared fault gouge: Implications of acoustic vibration on triggering and slow slip - Illinois, accessed May 12, 2026, http://publish.illinois.edu/mcslabuiuc/files/2017/12/2016-Lieou-Elbanna-Carlson.pdf
34. The emergence of crack-like behavior of frictional rupture: The origin of stress drops | Request PDF - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/334082331_The_emergence_of_crack-like_behavior_of_frictional_rupture_The_origin_of_stress_drops
35. Solar oscillations and helioseismology, accessed May 12, 2026, https://www.mssl.ucl.ac.uk/~lvdg/PHAS_M314_osc_11.pdf
36. Helioseismology - Wikipedia, accessed May 12, 2026, https://en.wikipedia.org/wiki/Helioseismology
37. arXiv:astro-ph/9612120v1 11 Dec 1996, accessed May 12, 2026, https://arxiv.org/pdf/astro-ph/9612120
38. 8. Solar oscillations, accessed May 12, 2026, http://sun.stanford.edu/~sasha/PHYS780/SOLAR_PHYSICS/L8/Lecture_08_PHYS78

0.pdf
39. Assessment of DKIST/VTF capabilities for the detection of local acoustic source wavefronts, accessed May 12, 2026, https://www.frontiersin.org/journals/astronomy-and-space-sciences/articles/10.3389/fspas.2026.1797722/full
40. What is the difference between manifold learning and non-linear dimensionality reduction?, accessed May 12, 2026, https://stats.stackexchange.com/questions/247907/what-is-the-difference-between-manifold-learning-and-non-linear-dimensionality-r
41. GENERATIVE MODELING BASED MANIFOLD LEARNING FOR ADAPTIVE FILTERING GUIDANCE Karim Helwani∗, Paris Smaragdis∗,], and Michael - Amazon Science, accessed May 12, 2026, https://assets.amazon.science/52/76/a228a6534068937d7c29ad478ac0/generative-modeling-based-manifold-learning-for-adaptive-filtering-guidance.pdf
42. Neurospectrum: A Geometric and Topological Deep Learning Framework for Uncovering Spatiotemporal Signatures in Neural Activity - PMC, accessed May 12, 2026, https://pmc.ncbi.nlm.nih.gov/articles/PMC12247765/
43. (PDF) Learning a potential formulation for rate-and-state friction - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/393685010_Learning_a_potential_formulation_for_rate-and-state_friction
44. Learning a potential formulation for rate-and-state friction | Request PDF - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/397412945_Learning_a_potential_formulation_for_rate-and-state_friction
45. accessed December 31, 1969, https://arxiv.org/abs/2507.09796
46. Experimental verification of rate-dependent elastoplastic analogy friction model and its application to FE analysis - ResearchGate, accessed May 12, 2026, https://www.researchgate.net/publication/257127950_Experimental_verification_of_rate-dependent_elastoplastic_analogy_friction_model_and_its_application_to_FE_analysis